Model Reduction for Nonlinear Evolutionary Dynamics

Andreas Degenhard
Fakultät für Physik, Universität Bielefeld, Germany
Javier Rodriguez Laguna
IFT (UAM - CSIC) Universidad Autnoma de Madrid, Madrid, Spain

We propose a theoretical and computational approach to calculate the relevant degrees of freedom for systems out of equilibrium using a multiscale technique. As particular examples we examine the evolution governed by non-linear partial differential equations such as the Burgers' equation with impact on hydrodynamics or the KPZ-equation describing the physics of surface growth phenomena. Other schemes for coarse-graining systems far from equilibrium do exist however, in general, rely on perturbative approaches and are therefore not universally applicable.

The proposed multiscale approach is formulated within a rigorous, analytic framework. To test the approach results were compared to exact solutions available for linear problems demonstrating complete agreement. Our technique allows modelling the evolutionary process with a reduced number of degrees of freedom. The method is applicable within any temporal regime of interest and various types of nonlinear dynamical behaviour are analyzed. This is demonstrated by examining the related scaling characteristics of the system.

The method is formulated in the context of projection operators used in modern renormalization group (RG) techniques. Therein an embedding and a truncation operator are constructed to formulate the so called renormalization group transformation (RGT) governing the reduction process. For applications within equilibrium the construction of these operators usually involves the low energy spectrum to define an orthonormal basis system for the relevant subspace. Although using the same formalism the construction of our embedding and truncation operators for non-equilibrium processes is completely different.

Our method is very close to the original RG idea of L.P. Kadanoff and K.G. Wilson in that it resembles their blockspin concept within a generalized coarse graining approach. This approach allows for incorporating correlation effects between adjacent blocks, i.e. inter-block correlations within the blocked system, into the (block spin) RGT. Currently we compare our approach to standard numerical multiscale schemes allowing to determine reduced orthogonal basis systems. This includes, for example, various wavelet decomposition techniques as for example used in the scale decoposition of the Burgers' equation.