Constructive Methods of Invariant Manifolds for Kinetic Problems

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We present the Constructive Methods of Invariant Manifolds for model reduction in physical and chemical kinetics, developed during last two decades. The physical problem of reduced description is studied in a most general form as a problem of constructing the slow invariant manifold. The invariance conditions are formulated as the differential equation for a manifold immersed in the phase space (*the invariance equation*). The equation of motion for immersed manifolds is obtained (*the film extension of the dynamics*). Invariant manifolds are fixed points for this equation, and slow invariant manifolds are Lyapunov stable fixed points, thus *slowness is presented as stability.* A collection of methods for construction of slow invariant manifolds is presented, in particular, the Newton method subject to incomplete linearization is the analogue of *KAM methods for dissipative systems.* The systematic use of thermodynamics structures and of the quasi–chemical representation allow to construct approximations which are in concordance with physical restrictions. We systematically consider a discrete analogue of the slow (stable) positively invariant manifolds for dissipative systems, *invariant grids.* Dynamic and static *postprocessing procedures* give us the opportunity to estimate the accuracy of obtained approximations, and to improve this accuracy significantly.

The following examples of applications are presented: Nonperturbative deviation of physically consistent hydrodynamics from the Boltzmann equation and from the reversible dynamics, for Knudsen numbers $Kn \sim 1$; construction of the moment equations for nonequilibrium media and their dynamical correction (instead of extension of list of variables) to gain more accuracy in description of highly nonequilibrium flows; determination of molecules dimension (as diameters of equivalent hard spheres) from experimental viscosity data; invariant grids for a two–dimensional catalytic reaction and a four–dimensional oxidation reaction (six species, two balances); universal continuous media description of

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dilute polymeric solution; the limits of macroscopic description for polymer molecules, etc.

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