Singular Invariance PDEs for Nonlinear Skew Product Systems With Application to a Molecular Photodissociation Control Problem

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The present work aims at the development of a systematic framework for the derivation of model-based control laws that attain the invariance objective for nonlinear input-driven skew product dynamical systems. The problem under consideration naturally arises in a variety of control problems pertaining to physical/chemical systems and processes, and in the present study, it is conveniently formulated and addressed in the context of singular PDE theory. In particular, the mathematical formulation of the problem of interest is realized via a system of first-order quasi-linear singular PDEs and a rather general set of necessary and sufficient conditions for solvability is explicitly derived. The solution to the above system of singular PDEs can be proven to be a unique locally analytic one, and this enables the development of a series solution method that is easily programmable with the aid of a symbolic software package such as MAPLE. It is also shown, that on the basis of the solution to the above system of PDEs, a locally analytic manifold and a nonlinear control law can be explicitly derived that renders the manifold invariant. Furthermore, the restriction of the system dynamics on the aforementioned invariant manifold represents exactly the target/desired controlled system dynamics. Finally, the proposed method is applied to the HF molecular system classically modeled as a rotationless Morse oscillator in the presence of an external laser-field, where the primary objective is molecular dissociation.