Collapse in Hydrodynamics and Kolmogorov Spectrum

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Abstract

To describe the vortical flows of ideal fluids including collapse we suggest a mixed Lagrangian-Eulerian description. It is shown that the Euler hydrodynamics in the new representation coincides with equations of motion for a charged liquid flowing due to the Lorents force in an electromagnetic field [1]. As a consequence of such representation it is possible to construct the analog of the Weber transformation and to find a new Cauchy invariant that is a key point for investigation of the collapse in hydrodynamics. A new mechanism of the collapse in hydrodynamics is suggested, due to breaking of continuously distributed vortex lines. Collapse results in formation of the point singularities of the vorticity field Ω. At the collapse point, the value of the vorticity blows up as $(t_0-t)^{-1}$ where t_0 is a collapse time. The spatial structure of the collapsing distribution approaches a pancake form: contraction occurs by the law $l_1 \sim (t_0-t)^{3/2}$ along the "soft" direction, the characteristic scales vanish like $l_2 \sim (t_0-t)^{1/2}$ along two other ("hard") directions. This scenario of the collapse is shown to take place in the integrable three-dimensional hydrodynamics with the Hamiltonian $\mathcal{H} = \int |\Omega| dr$. Within the 3D Euler equations, which are resolved relative to the infinite set of the Cauchy invariants, emergence of singularity of vorticity at a single point not related to any symmetry of the initial distribution, has been demonstrated numerically for the first time [2]. Behavior of the maximum of vorticity near the point of collapse closely follows the dependence $(t_0-t)^{-1}$. Sequences of such type of collapse are discussed for fully developed hydrodynamic turbulence. In particular, it is demonstrated that structure of vorticity near the breaking point has the Kolmogorov behavior.

References

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