The methods of Chapman-Enskog and Grad, revisited

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Abstract

In this talk some recent results on approximate methods in the kinetic theory of gases will be reported.

The ES-BGK model equation allows to perform the Chapman-Enskog expansion for a model gas with freely varying Prandtl number. It will be shown that for certain values of the Prandtl number Bobylev's well known instability of the Burnett equations (i.e. the second order CE-expansion) does not occur. It follows that higher order Chapman-Enskog expansions do not necessarily lead to unstable equations.

Then a simple argument is presented to show that information present in the kinetic equation is lost when the Chapman-Enskog expansion is performed, but not when Grad's moment method is performed.

Finally, we present a new method for regularization of Grad's 13 moment equations, which adds terms of Super-Burnett order to the balances of pressure deviator and heat flux vector. The additional terms are derived from equations for higher moments by means of the distribution function for 13 moments. The resulting system of equations contains the Burnett and Super-Burnett equations when expanded in a series in the Knudsen number. However, other than the Burnett and Super-Burnett equations, the new set of equations is linearly stable for all wavelengths and frequencies. Dispersion relation and damping for the new equations agree better with experimental data than those for the Navier-Stokes-Fourier equations, or the original 13 moments system. The new equations also allow the description of Knudsen boundary layers. Moreover, due to additional higher order dissipation in the system shock structure profiles are smooth for any Mach number in contrast to the results of Grad's 13-moment-case. The results show satisfying agreement with measurements and DSMC simulations. Nevertheless the theory is still restricted to Mach numbers up to Ma$\simeq$2.5, which is an improvement over classical theories.