INVARIANT MANIFOLDS OF INVERTIBLE MAPS AND LOCALIZED OSCILLATIONS IN NONLINEAR LATTICES

Tassos Bountis
Jeroen Bergamin
Department of Mathematics and
Center of Research and Applications of Nonlinear Systems
University of Patras
Patras 26500 HELLAS (Greece)

Discrete breathers, or localized periodic oscillations of nonlinear lattices, are an important phenomenon affecting the propagation of energy in various physical systems. In this work, we first use Fourier analysis and derive nonlinear recurrence relations (maps) with a saddle fixed point at the origin to establish a connection between discrete breathers and the homoclinic intersections of the invariant manifolds associated with that point. Since the recurrence relations satisfied by the Fourier coefficients generally involve infinitely many modes, we study reductions to two-dimensional invertible maps and obtain surprisingly accurate approximations of the homoclinic solutions of the infinite dimensional map. The reduction to maps, however, in Fourier amplitude space is limited to one-dimensional lattices in which the particles experience polynomial on-site potentials and interact linearly with their nearest neighbors. Thus, we have separated approximately the temporal from the spatial part of the solution and derived a system of second order ODEs for the temporal oscillations which becomes overdetermined, when we demand that its solutions have a specified period. This leads again to a two-dimensional invertible map that yields the breather solutions through the homoclinic intersections of its invariant manifolds. The approximations thus obtained can now be carried out for a much wider class of nonlinearities to compute breathers in realistc lattices of more than one spatial dimension.