From Physics to Robust Control of Dissipative Systems

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Dissipation is a physical concept closely related to the first and the second law of thermodynamics. The first law ensures conservation of mass and energy in all its forms. The second law determines the way in which the different forms of energy and material species *evolve* (relate and convert one into each other) through transport phenomena and chemical reactions, taking place on a given spatial domain. The evolution criterion is formally stated in terms of a concave function, called entropy, which never decreases in isolated processes and achieves its maximum at equilibrium. Thus, systems out of equilibrium spontaneously evolve to equilibrium through irreversible processes that produce entropy. In this way, dissipation is a positive function that quantifies the rate at which entropy is produced (Glansdorff and Prigogine, 1971).

This notion remains valid for open systems, we will refer to as *Dissipative Systems*. Now, in addition to entropy production, there exists an entropy flux between the system and its surroundings as materials and energy flow through the domain. In this context, dissipation imposes a particular relationship between transport processes and their associated thermodynamic forces (gradients) which guides the dynamic evolution of the system. However, the combined action of fluxes and rate processes can move the states of the system far away from equilibrium thus giving room to a rich variety of complex behaviours. From a control perspective, understanding the interplay between fluxes and dissipation seems essential to guide (control) the evolution of dissipative systems. Such objective was stated in 1934 by Donnan and Guggenheim in the following terms:

"A finite amount of organization may be purchased at the expense of a greater amount of disorganization in a series of interrelated spontaneous actions"

With the intention of developing efficient ways of *purchasing organization*, we explore connections between the underlying physics of dissipative systems and non-linear robust control. In particular, we concentrate on the problem of stabilizing stationary solutions of non-linear dissipative systems with states distributed in space (*Distributed Process Systems*). This class of dissipative systems plays a central role in many biological systems as well as in chemical and material processing industries, as many of its operations involve convection diffusion and reaction phenomena. Interesting examples include, to name a few, catalytic reactors, chemical vapour deposition units, crystallization or thermal processing.

The control of distributed process systems has received considerable attention from the control community over the last years. Standard approaches rely on a state-space-like representation of the original infinite dimensional system by spatial discretization of the set of partial differential equations. Common discretization schemes include finite differences or finite elements. Standard linear or non-linear finite dimensional control design methods are then employed to construct the controller. Alternative control design methods, which take into account the spatially distributed nature of the system, are based on spectral decomposition schemes which retains the essential properties of the spatial differential operator.

A complementary approach makes use of the second law of thermodynamics and passivity, as it is understood in systems theory. The second law, in the exergy form, gives convexity, which in turns provides a general answer to the question of finding Lyapunov function candidates to assess system's evolution. Passivity concepts link inputs to outputs while preserving the infinite dimensional structure of the system.

In this work, we maintain such thermodynamic formalism to explore new links between the underlying physics of dissipative process systems and non-linear control. The existence of an entropy-like function will allow us to relate dissipation with a Hamilton-Jacobi-Bellman type equation. Such connection will open direct ways to establish passivity conditions for dissipative systems. In this regard, one main conclusion is that any dissipative system is in fact passive when appropriate inputs and outputs are selected. We also derive an optimal stabilizing control result which can be considered as a general re-statement of *Prigogine's Minimum Entropy Production* principle. In the light of these results, we finally discuss robustness issues in controller design, as it may become a relevant problem in the control of front or pulse-type spatial pattern formation.